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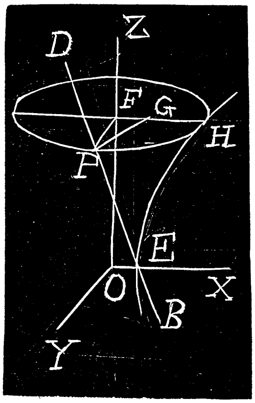
# THE HYPERBOLOID AS A RULED SURFACE.*

By JAMES PERRY WILSON, Columbia University.

If a straight line is revolved around another line not in the same plane, as indicated below, it will generate a hyperboloid of one sheet.



Take the $Z$-axis as the fixed line. We can then fix the other axes so that the generating line, in its initial position $BD$, will be parallel to the $YZ$-plane, and will intersect the $X$-axis at $E$. The equations of the generating line in this position will then be of the form $x=a$, $y=mz$.

As the line is revolved around the $Z$-axis, every point on the line will generate a circle around the $Z$-axis; and the locus of the intersections of these circles with the $XZ$-plane will indicate the nature of the surface generated by the line.

Consider any point $P(a, mz, z)$ on the line $BD$. As the line is revolved around $OZ$, $P$ will describe a circle around $OZ$, whose radius $FP=\sqrt{(FG^2+GP^2)}=\sqrt{(a^2+m^2z^2)}$. This circle will intersect the $XZ$-plane at $H$; then

$$FH=x=\sqrt{(a^2+m^2z^2)} \ldots (1).$$

Since $P$ is any point on the generating line, in its initial position, equation (1) represents the locus of the intersections with the $XZ$-plane, of the circles generated by all the points on the moving line.

From (1), by rationalizing, transposing, and dividing by $a^2$, we obtain

$$\frac{x^2}{a^2}-\frac{m^2z^2}{a^2}=1.$$

Putting $\frac{a^2}{m^2}=c^2$, we obtain

$$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1 \ldots (2),$$

which is the standard equation of the hyperbola; and the surface generated by revolving $BD$ around $OZ$ is the same as that generated by revolving the hyperbola (2) around $OZ$, its conjugate axis; that is, the hyperboloid

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*This note was presented by a student in a beginner's class in solid analytical geometry. Its extreme simplicity seems to warrant its publication.

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=1.$$

If the moving line is perpendicular to the fixed line, it will lie wholly in the plane $Z=k$, and the hyperboloid will degenerate into the plane figure $Z=k$, $x^2+y^2 \geqq a^2$, $a$ in this case being the shortest distance between the lines $OZ$ and $BD$.

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## THE SOLUTION OF AN EQUATION BY A FRAME.

**By T. M. BLAKSLEE, Ames, Iowa.**

The *frame* of the equation

$$f(x)=x^n+bx^{n-1}+cx^{n-2}+dx^{n-3}+\ldots=0,$$

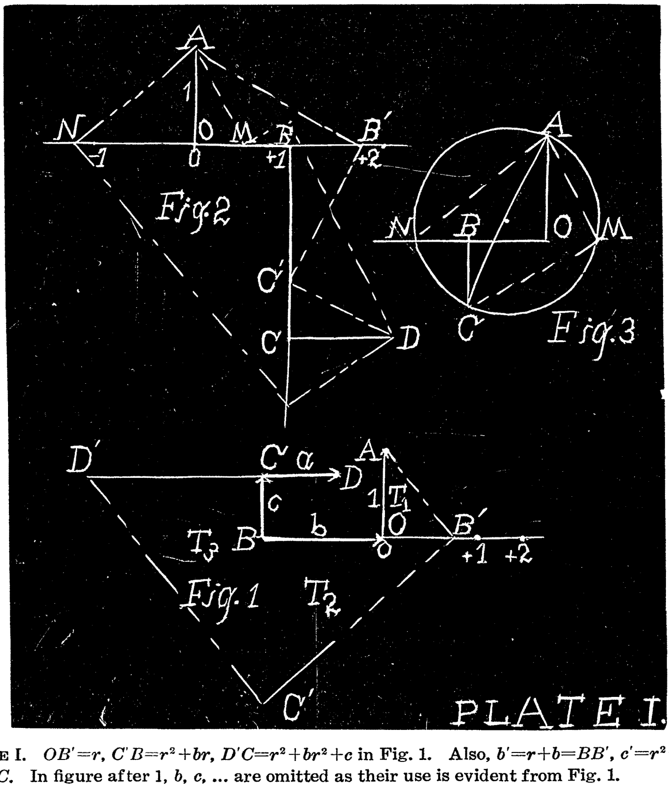

PLATE I. $OB'=r$, $C'B=r^2+br$, $D'C=r^2+br^2+c$ in Fig. 1. Also, $b'=r+b=BB'$, $c'=r^2+br+c=C'C$. In figure after 1, $b$, $c$, ... are omitted as their use is evident from Fig. 1.